PERGAMON

International Journal of Heat and Mass Transfer 45 (2002) 2067–2075

International Journal of
HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

A decomposition method for solving the convective longitudinal fins with variable thermal conductivity

Ching-Huang Chiu, Cha'o-Kuang Chen *

Department of Mechanical Engineering, National Cheng Kung University, Tainan 70101, Taiwan, ROC

Received 5 January 2001; received in revised form 26 August 2001

Abstract

In this paper the Adomian decomposition method is used to evaluate the efficiency and the optimal length of a convective rectangular fin with variable thermal conductivity, and to determine the temperature distribution within the fin. It is a useful and practical method, which can be used to solve the nonlinear energy balance equations which are associated with variable thermal conductivity conditions. The Adomian decomposition method provides an analytical solution in the form of an infinite power series. From a practical perspective, it is necessary to evaluate this analytical solution, and to obtain numerical values from the infinite power series. This requires series truncation, and a practical procedure to accomplish the task. Together, these transform the analytical results into a solution with a finite degree of accuracy. The accuracy of the Adomian decomposition method with a varying number of terms in the series is investigated by comparing its results with those generated by a finite-difference method which uses a Newton linearization scheme. © 2002 Published by Elsevier Science Ltd.

Keywords: Adomian decomposition method; Nonlinear differential equations; Convective fin; Optimization

1. Introduction

Fins are widely used to enhance the heat transfer between a solid surface and its convective, radiative, or convective–radiative surface [1]. A considerable amount of research has been conducted into the variable thermal parameters which are associated with fins operating in practical situations. For example, Razelos and Imre [2] considered the variation of the convective heat transfer coefficient from the base of a convecting fin to its tip. If a large temperature difference exists within the fin, the dependence of the thermal conductivity of the fin on the temperature can be significant. It would seem that the study of heat transfer in fins with variable thermal parameters is in order. Hung and App [3] presented the performance of a straight fin with temperature-dependent conductivity and internal heat generation, while Jany and Bejan investigated the optimum shape for straight fins with temperature-dependent conductivity [4]. The governing equation of fins with temperature-dependent conductivity is in the form of a nonlinear differential equation, and, in most cases, solving these equations involves numerical procedures. Aziz [5] and Krane [6] used the regular perturbation method and a numerical solution method to present a closed form solution for a straight convecting fin with temperature-dependent thermal conductivity, while an alternative approach based on the Galerkin method was used by Muzzio [7] to obtain approximate analytical solutions.

Recently, the Adomian decomposition scheme [8] has emerged as an alternative method for solving a wide range of problems whose mathematical models involve algebraic [9], differential [10], integro-differential [11], and partial differential equations [12]. The decomposition method yields rapidly convergent series solutions for both linear and nonlinear deterministic and stochastic equations. The advantage of this method is that it provides a direct scheme for solving the problem, i.e., without the need for linearization.

This paper applies the Adomian decomposition method to a nonlinear conduction–convection heat

* Corresponding author. Tel.: +886-6-275-7575; fax: +886-6-234-2081.

E-mail address: ckchen@mail.ncku.edu.tw (C.-K. Chen).

0017-9310/02/$ - see front matter © 2002 Published by Elsevier Science Ltd.
PII: S0017-9310(01)00286-1

Nomenclature

A_c	cross sectional area of the fin
A_m	Adomian's polynomial
A_p	profile area of the fin
b	fin length
C	integral constant
h	heat transfer coefficient
k	thermal conductivity
L	the highest order derivative
L^{-1}	inverse operator of L
N	dimensionless fin parameter $[Phb^2/k_aA_c]^{1/2}$
P	fin perimeter
T	temperature
N_u	nonlinear terms
q_f	heat transfer rate
Q_n	dimensionless heat transfer
V	volume of the fin
w	fin thickness
x	axial distance measured from fin tip
X	dimensionless distance x/b

Greek symbols

β	slop of the thermal conductivity–temperature divided by the intercept k_a
ε	thermal conductivity parameter $(k_b - k_a)/k_a$
θ	dimensionless temperature $(T - T_a)/(T_b - T_a)$
η	fin efficiency

Subscripts

a	ambient
b	base of the fin
e	tip of the fin
dec.	decomposition solution
m	number of terms in the series
num.	numerical solution
opt	optimal

transfer equation to derive an approximate analytical solution. The results of the decomposition method are then compared with those obtained from a numerical solution, the perturbation solution and the Galerkin approximate solution.

2. The governing equation and boundary condition

A rectangular fin with length, b, and thickness, w, is considered. The fin tip is assumed to be insulated and a uniform temperature is assumed at the fin base. Under steady-state conditions, the faces of the fin are exposed to a convective environment where the temperature, T_a, and heat transfer coefficient, h, are assumed to be uniform. Fig. 1 shows an illustration of the fin geometry, where the axial distance, x, is measured from the fin tip. The thermal conductivity of the fin material, k, is assumed to vary as a linear function of the temperature, i.e.

$$k = k_a[1 + \beta(T - T_a)], \tag{1}$$

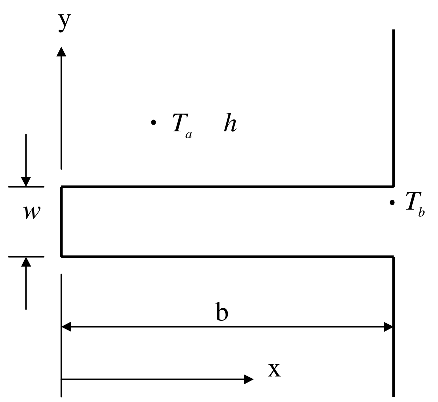

Fig. 1. Schematic diagram of a rectangular longitudinal fin.

where k_a is the thermal conductivity at ambient temperature, and β is the slope of the thermal conductivity–temperature curve divided by the intercept, k_a.

If the Biot number, hw/k, of the fin is less than 0.1, then the effect of heat conduction in the y-direction on the rate of heat transfer from the fin appears to be quite small (i.e., less than 1%) [13]. In the one-dimensional system, the energy balance equation and the aforesaid boundary conditions are:

$$(1 + \varepsilon\theta)\frac{d^2\theta}{dx^2} + \varepsilon\left(\frac{d\theta}{dx}\right)^2 - N^2\theta = 0, \tag{2}$$

$$X = 0, \quad \frac{d\theta}{dx} = 0, \tag{3}$$

$$X = 1, \quad \theta = 1, \tag{4}$$

where

$$\theta = \frac{T - T_a}{T_b - T_a}, \quad X = \frac{x}{b}, \quad \varepsilon = \frac{k_b - k_a}{k_a} = \beta(T_b - T_a),$$
$$N^2 = \frac{Phb^2}{k_aA_c} = \frac{2hb^2}{k_aw}. \tag{5}$$

3. Decomposition method

The principal algorithm of the Adomian decomposition method when applied to a general nonlinear equation is in the form

$$Lu + Ru + Nu = g. \tag{6}$$

The linear terms are decomposed into $L+R$, while the nonlinear terms are represented by Nu. L is taken as the highest order derivative to avoid difficult integration involving complicated Green's functions, and R is the remainder of the linear operator. L^{-1} is regarded as the inverse operator of L and is defined by a definite integration from 0 to x, i.e.

$$[L^{-1}f](x) := \int_0^x f(v)\,\mathrm{d}v. \tag{7}$$

If L is a second-order operator, L^{-1} is a twofold indefinite integral, i.e.

$$L^{-1}Lu = u - u(0) - x\frac{\partial u(0)}{\partial x}. \tag{8}$$

Operating on both sides of Eq. (6) with L^{-1} yields

$$L^{-1}Lu = L^{-1}g - L^{-1}Ru - L^{-1}Nu \tag{9}$$

and gives

$$u = u(0) + x\frac{\partial u(0)}{\partial x} + L^{-1}g - L^{-1}Ru - L^{-1}Nu. \tag{10}$$

The decomposition technique represents the solution of Eq. (9) as a series, i.e., $u=\sum_{m=0}^{\infty}u_m$. The nonlinear operator, Nu, is decomposed as $\sum_{m=0}^{\infty}A_m$.

$$\sum_{m=0}^{\infty}u_m = u_0 - L^{-1}R\sum_{m=0}^{\infty}u_m - L^{-1}\sum_{m=0}^{\infty}A_m, \tag{11}$$

where

$$u_0 = u(0) + x\frac{\partial u(0)}{\partial x} + L^{-1}g.$$

Consequently it can be written as

$$\begin{aligned} u_1 &= -L^{-1}Ru_0 - L^{-1}A_0,\\ u_2 &= -L^{-1}Ru_1 - L^{-1}A_1,\\ &\vdots\\ u_{m+1} &= -L^{-1}Ru_m - L^{-1}A_m, \end{aligned} \tag{12}$$

where A_m's are Adomian's polynomial of $u_0, u_1, \ldots, u_m$, and are obtained from the formula

$$A_m = \frac{1}{m!}\frac{\mathrm{d}^m}{\mathrm{d}\lambda^m}[f(u(\lambda))]_{\lambda=0.} \tag{13}$$

Eq. (13) gives

$$\begin{aligned} A_0 &= f(u_0),\\ A_1 &= \left(\frac{\mathrm{d}f}{\mathrm{d}u}\right)\left(\frac{\mathrm{d}u}{\mathrm{d}\lambda}\right)\Big|_{\lambda=0},\\ A_2 &= \frac{1}{2!}\left[\left(\frac{\mathrm{d}^2f}{\mathrm{d}u^2}\right)\left(\frac{\mathrm{d}u}{\mathrm{d}\lambda}\right)^2 + \left(\frac{\mathrm{d}f}{\mathrm{d}u}\right)\left(\frac{\mathrm{d}^2u}{\mathrm{d}\lambda^2}\right)\right]\Big|_{\lambda=0},\\ A_3 &= \frac{1}{3!}\left[\left(\frac{\mathrm{d}^3f}{\mathrm{d}u^3}\right)\left(\frac{\mathrm{d}u}{\mathrm{d}\lambda}\right)^3 + 2\left(\frac{\mathrm{d}^2f}{\mathrm{d}u^2}\right)\left(\frac{\mathrm{d}u}{\mathrm{d}\lambda}\right)\left(\frac{\mathrm{d}^2u}{\mathrm{d}\lambda^2}\right)\right.\\ &\quad\left. + \left(\frac{\mathrm{d}^2f}{\mathrm{d}u^2}\right)\left(\frac{\mathrm{d}^2u}{\mathrm{d}\lambda^2}\right)\left(\frac{\mathrm{d}u}{\mathrm{d}\lambda}\right) + \left(\frac{\mathrm{d}f}{\mathrm{d}u}\right)\left(\frac{\mathrm{d}^3u}{\mathrm{d}\lambda^3}\right)\right]\Big|_{\lambda=0},\\ &\vdots \end{aligned}$$

Finally, the A_m's can be written in the following, more convenient, form [14]:

$$A_m = \sum_{v=1}^{m} c(v,m)f^{(v)}(u_0), \quad m \geqslant 1, \tag{14}$$

where $c(v,m)$ are products of the v components of u, whose subscripts sum to m, divided by the factorial of the number of repeated subscripts. Thus, the A_m's are expressed as:

$$\begin{aligned} A_0 &= f(u_0),\\ A_1 &= u_1\frac{\mathrm{d}}{\mathrm{d}u_0}f(u_0),\\ A_2 &= u_2\frac{\mathrm{d}}{\mathrm{d}u_0}f(u_0) + \frac{u_1^2}{2!}\frac{\mathrm{d}^2}{\mathrm{d}u_0^2}f(u_0),\\ A_3 &= u_3\frac{\mathrm{d}}{\mathrm{d}u_0}f(u_0) + u_1u_2\frac{\mathrm{d}^2}{\mathrm{d}u_0^2}f(u_0) + \frac{u_1^3}{3!}\frac{\mathrm{d}^3}{\mathrm{d}u_0^3}f(u_0),\\ &\vdots \end{aligned}$$

4. The fin temperature distribution

Following Adomian decomposition analysis, the linear operator is defined as: $L_x = \mathrm{d}^2\theta/\mathrm{d}X^2$. Consequently Eq. (2) can be written as follows:

$$\begin{aligned} L_x\theta &= N^2\theta - \varepsilon\theta\frac{\mathrm{d}^2\theta}{\mathrm{d}X^2} - \varepsilon\left(\frac{\mathrm{d}\theta}{\mathrm{d}X}\right)^2\\ &= N^2\theta - \varepsilon NA - \varepsilon NB, \end{aligned} \tag{15}$$

where

$$NA = \theta\frac{\mathrm{d}^2\theta}{\mathrm{d}X^2} = \sum_{m=0}^{\infty}A_m,$$

$$NB = \left(\frac{\mathrm{d}\theta}{\mathrm{d}X}\right)^2 = \sum_{m=0}^{\infty}B_m$$

are nonlinear terms. Hence, using Eq. (14) gives

$$\begin{aligned} A_0 &= \theta_0\frac{\mathrm{d}^2\theta_0}{\mathrm{d}X^2},\\ A_1 &= \theta_1\frac{\mathrm{d}^2\theta_0}{\mathrm{d}X^2} + \theta_0\frac{\mathrm{d}^2\theta_1}{\mathrm{d}X^2},\\ A_2 &= \theta_2\frac{\mathrm{d}^2\theta_0}{\mathrm{d}X^2} + \theta_1\frac{\mathrm{d}^2\theta_1}{\mathrm{d}X^2} + \theta_0\frac{\mathrm{d}^2\theta_2}{\mathrm{d}X^2},\\ A_3 &= \theta_3\frac{\mathrm{d}^2\theta_0}{\mathrm{d}X^2} + \theta_2\frac{\mathrm{d}^2\theta_1}{\mathrm{d}X^2} + \theta_1\frac{\mathrm{d}^2\theta_2}{\mathrm{d}X^2} + \theta_0\frac{\mathrm{d}^2\theta_3}{\mathrm{d}X^2},\\ &\vdots \end{aligned} \tag{16}$$

and

$$
\begin{aligned}
B_0 &= \left(\frac{\mathrm{d}\theta_0}{\mathrm{d}X}\right)^2, \\
B_1 &= 2\frac{\mathrm{d}\theta_0}{\mathrm{d}X}\frac{\mathrm{d}\theta_1}{\mathrm{d}X}, \\
B_2 &= \left(\frac{\mathrm{d}\theta_1}{\mathrm{d}X}\right)^2 + 2\frac{\mathrm{d}\theta_0}{\mathrm{d}X}\frac{\mathrm{d}\theta_2}{\mathrm{d}X}, \\
B_3 &= 2\frac{\mathrm{d}\theta_1}{\mathrm{d}X}\frac{\mathrm{d}\theta_2}{\mathrm{d}X} + 2\frac{\mathrm{d}\theta_0}{\mathrm{d}X}\frac{\mathrm{d}\theta_3}{\mathrm{d}X}, \\
&\vdots
\end{aligned}
\tag{17}
$$

Operating on both sides of Eq. (15) with L_x^{-1} yields

$$L_x^{-1}L_x\theta = N^2L_x^{-1}\theta - \varepsilon L_x^{-1}NA - \varepsilon L_x^{-1}NB, \tag{18}$$

$$\theta = \theta_0 + N^2L_x^{-1}\theta - \varepsilon L_x^{-1}NA - \varepsilon L_x^{-1}NB. \tag{19}$$

The value of the first term can be determined as

$$\theta_0 = \theta(0) + X\frac{\mathrm{d}\theta(0)}{\mathrm{d}X}. \tag{20}$$

With the boundary condition given in Eq. (3), $\theta(0)$ is any arbitrary constant, C.

The next iterates are determined from the following recursive relationship:

$$\theta_{m+1} = N^2L_x^{-1}\theta_m - \varepsilon L_x^{-1}A_m - \varepsilon L_x^{-1}B_m, \quad m \geqslant 0. \tag{21}$$

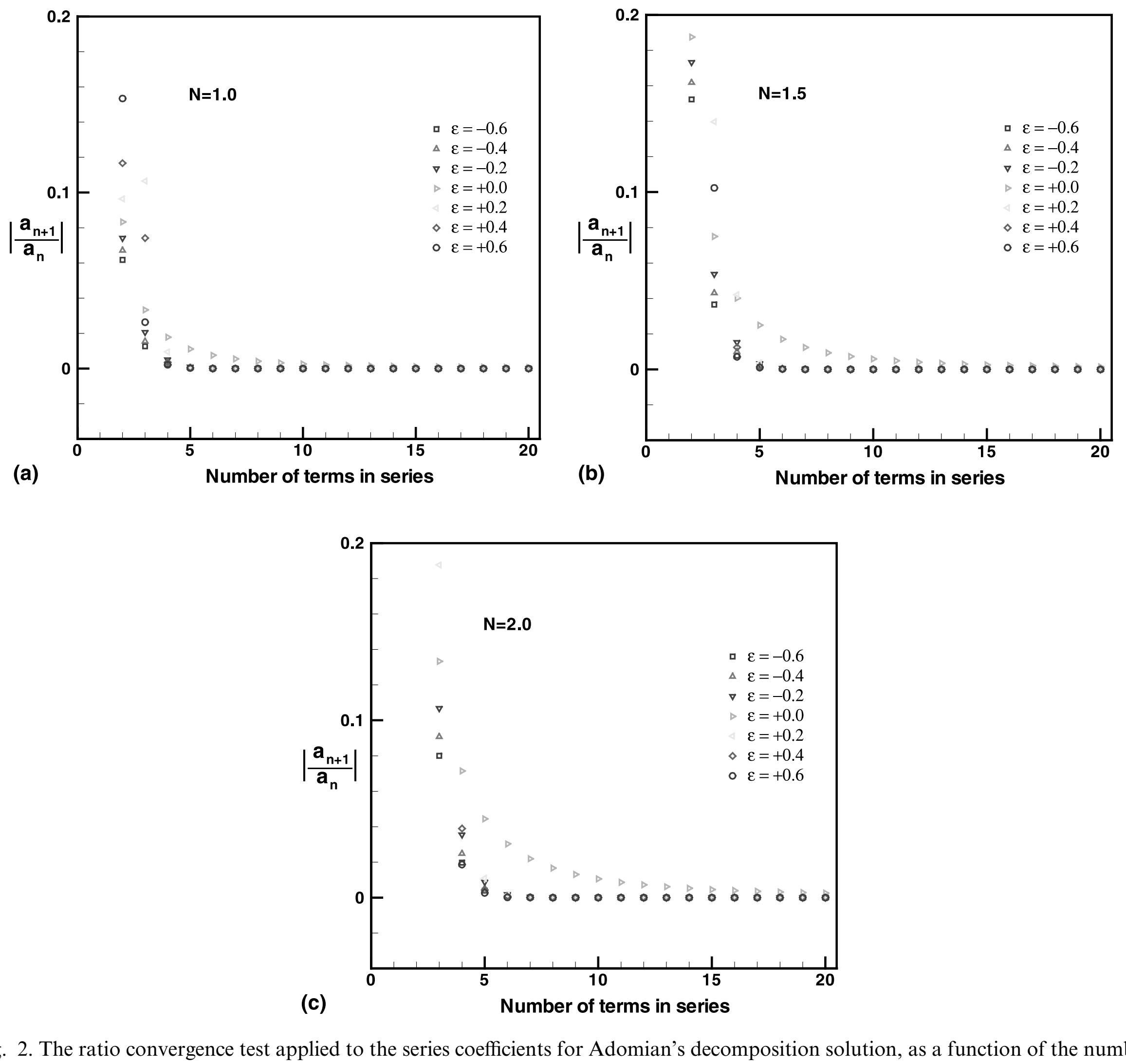

Fig. 2. The ratio convergence test applied to the series coefficients for Adomian's decomposition solution, as a function of the number of terms in the series, correspond to: (a) $N = 1.0$; (b) $N = 1.5$; (c) $N = 2.0$.

Therefore, the first four iterates are expressed as:

$$
\begin{aligned}
\theta_0 &= C,\\
\theta_1 &= N^2L_x^{-1}\theta_0 - \varepsilon L_x^{-1}A_0 - \varepsilon L_x^{-1}B_0 = \frac{1}{2}CN^2X^2,\\
\theta_2 &= N^2L_x^{-1}\theta_1 - \varepsilon L_x^{-1}A_1 - \varepsilon L_x^{-1}B_1\\
&= \frac{1}{24}CN^4X^4 - \frac{1}{2}\varepsilon C^2N^2X^2,\\
\theta_3 &= N^2L_x^{-1}\theta_2 - \varepsilon L_x^{-1}A_2 - \varepsilon L_x^{-1}B_2\\
&= -\frac{1}{12}\varepsilon C^2N^4X^4 - \varepsilon\left(\frac{1}{12}C^2N^4X^4 - \frac{1}{2}\varepsilon C^3N^2X^2\right)\\
&\quad + N^2\left(\frac{1}{720}CN^4X^6 - \frac{1}{24}\varepsilon C^2N^2X^4\right)\\
&\ \ \vdots
\end{aligned}
\tag{22}
$$

Upon summing those iterates it is observed that

$$
\varphi_m = \sum_{i=0}^{m-1}\theta_i = \theta_0 + \theta_1 + \theta_2 + \cdots + \theta_{m-1}. \tag{23}
$$

Thus, components of θ are determined and written as an m-terms approximation converging to θ as $m \to \infty$.

$$
\begin{aligned}
\theta = C &+ \frac{1}{2}CN^2X^2 + \frac{1}{24}CN^4X^4 - \frac{1}{2}\varepsilon C^2N^2X^2\\
&- \frac{1}{12}\varepsilon C^2N^4X^4 - \varepsilon\left(\frac{1}{12}C^2N^4X^4 - \frac{1}{2}\varepsilon C^3N^2X^2\right)\\
&+ N^2\left(\frac{1}{720}CN^4X^6 - \frac{1}{24}\varepsilon C^2N^2X^4\right)\cdots.
\end{aligned}
\tag{24}
$$

The coefficient, C, can be evaluated from the specific boundary condition given in Eq. (4). The available root

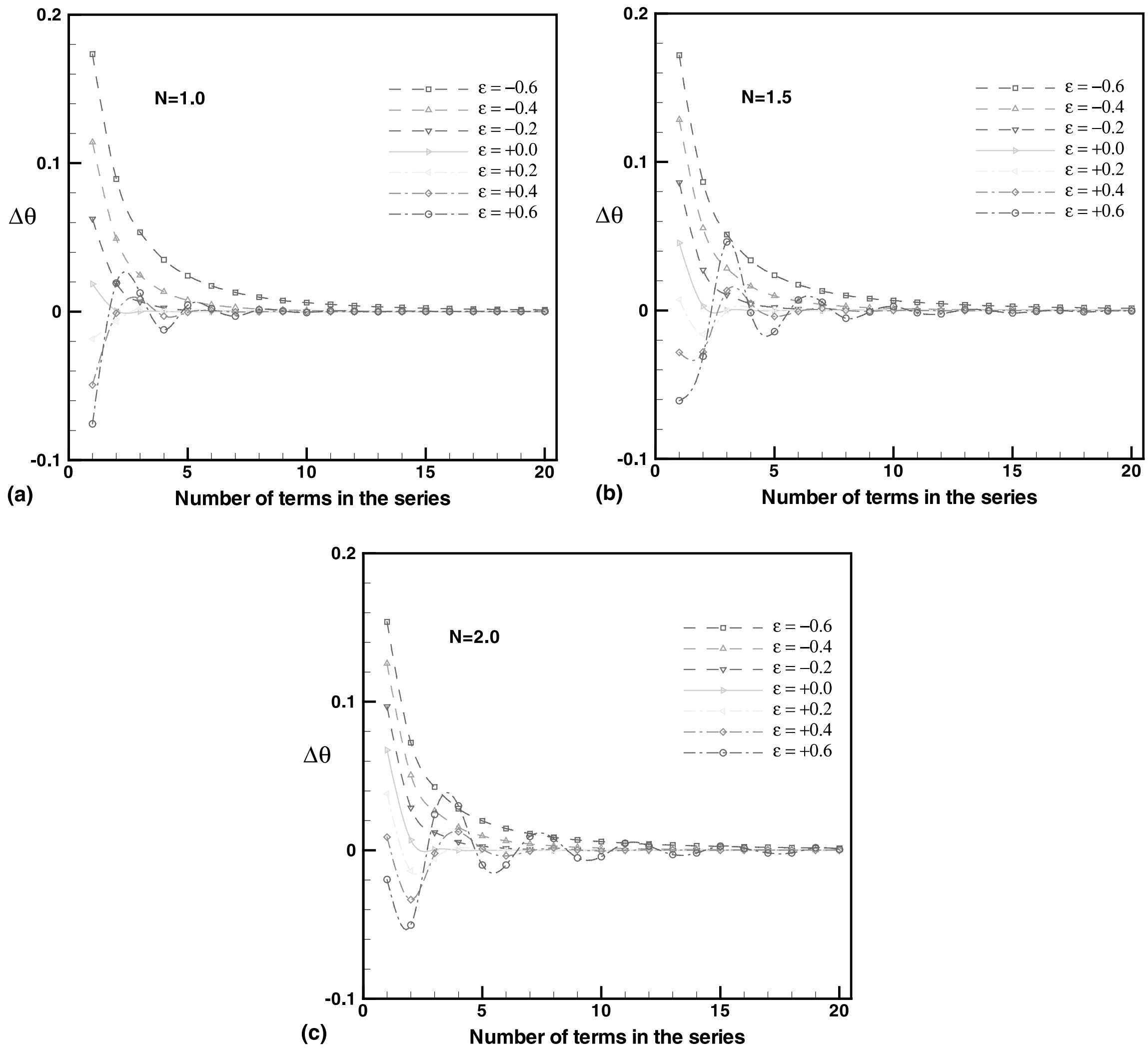

Fig. 3. The differences between the computational (Adomian's decomposition) and numerical solution at the tip temperature, as a function of the number of terms in the series, correspond to: (a) $N = 1.0$; (b) $N = 1.5$; (c) $N = 2.0$.

of C, used for representing the temperature at the fin tip, must lie in the interval $(0, 1)$; otherwise, it gives rise to physically meaningless temperature distributions and must be discarded.

5. Convergence of the series solution

The Adomian decomposition method provides an analytical solution in terms of an infinite power series. The analytical solution given in Eq. (24) can be expressed in the following series form:

$$\theta(X) = \sum_{m=0}^{\infty} a_m X^{2m}. \tag{25}$$

Re‘paci [15] provides rigorous proof of the convergence of the series solution for a more general form of the problem. The series (25) consists of both positive and negative terms, although not in a regular alternating fashion. The ratio test was applied to the absolute values of the series coefficient. This provides a sufficient condition for convergence of the series for a space interval $\Delta X = (X_b - X_e)^2$, in the form

$$\lim_{m\to\infty} \left| \frac{a_{m+1}}{a_m} \right| < \frac{1}{\Delta X}. \tag{26}$$

However, the approach which was preferred in this study to demonstrate the convergence of the series was to replace $\lim_{m\to\infty} |a_{m+1}/a_m|$ with $\lim_{m\to M} |a_{m+1}/a_m|$ in Eq. (26) where $M \to$ large constant. The behavior of the function $f(m) = |a_{m+1}/a_m|$ for increasing values of m was then observed. The results of the evaluated value of $f(m)$ for $m = 1, 2, \ldots, M$ (where $M = 20$) for $\varepsilon = -0.6$–0.6 corresponding to different values of N are presented in Figs. 2(a)–(c), respectively. It is clear from these figures that the ratio $f(m)$ decays as m increases, obviously indicating that the series (25) is convergent.

6. Accuracy of the Adomian decomposition solution

The Adomian decomposition method provides an analytical solution in the form of an infinite power series. However, there is a practical need to evaluate this solution, and to obtain numerical values from the infinite power series. The consequent series truncation, and the practical procedure conducted to accomplish this task, together transform the otherwise analytical results into a computational solution, which is evaluated to a finite degree of accuracy. In order to investigate the accuracy of the Adomian decomposition solution with a finite number of terms, Eq. (2) is also solved numerically, and the corresponding results are compared with the Adomian solution. The numerical method adopted in this study was the finite-difference scheme [16], which was used to discretize the nonlinear term. The Newton linearization scheme [17] was then applied to linearize the discretized result. The Adomian decomposition results were compared with the numerical solution by evaluating the difference between the temperatures given by the two methods at the fin tip and plotting this difference as $\Delta\theta = \theta_{\text{dec.}} - \theta_{\text{num.}}$, where $\theta_{\text{dec.}}$ and $\theta_{\text{num.}}$ represent the Adomian's decomposition and numerical results, respectively. The impact of the number of terms in the series solution, and the series truncation process, were assessed by evaluating the Adomian decomposition results for $\varepsilon = -0.6$–0.6 with 1–20 terms in the series, and then comparing them with the results of the numerical method. The results of the comparison for different N values are shown in Figs. 3(a)–(c), respectively. It can be observed from the figures that the shape of the difference remains quite small, of the order of magnitude 10^{-3}, as the number of terms, m, increases. For $\varepsilon = 0$, the difference is of the order of magnitude 10^{-5} for $m > 5$. As $m > 18$, the maximum difference never exceeds 0.002 for any value of ε. Therefore, it may be concluded that the use of 19 terms in the series yields sufficiently accurate results.

7. Fin efficiency and optimization

The heat dissipation of a fin can be obtained by integrating the convection heat loss from the fin surface, i.e.

$$q_f = \int_0^b Ph(T - T_a)\,\mathrm{d}x = b(T_b - T_a)\int_0^1 Ph\theta(X)\,\mathrm{d}X. \tag{27}$$

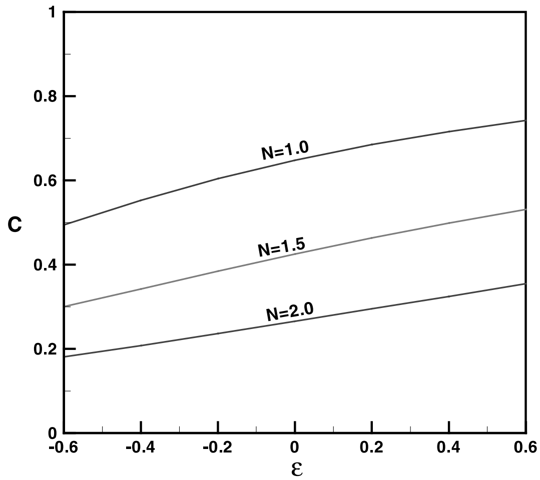

Fig. 4. Values of the coefficient C.

The efficiency of the fins is defined as the ratio of the actual heat transfer rate to the heat transfer rate of the entire fin surface, which depends on the temperature at the base of the fin, T_b. Thus

$$\eta = \frac{q_f}{Ph(T_b - T_a)} = \int_0^1 \theta(X)\,dX. \tag{28}$$

The fin volume is defined as: $V = A_c b$. Thus the heat dissipated per unit volume is

$$\frac{q_f}{V} = \frac{(T_b - T_a)\int_0^1 Ph\theta(X)\,dX}{A_c}. \tag{29}$$

The dimensionless heat transfer is defined as

$$Q_n = \left(\frac{q_f}{k_a(T_b - T_a)}\right)\left(\frac{A_p}{V}\right) = \frac{PhA_p}{k_a A_c}\int_0^1 \theta(X)\,dX = BN^{2/3}\int_0^1 \theta(X)\,dX, \tag{30}$$

$$A_p = wb, \quad B = \left(\frac{2h\sqrt{A_p}}{k_a}\right)^{2/3}.$$

The maximum heat dissipation value occurs at the condition when the optimum fin characteristics have been achieved. The fin dimensions in this situation represent the optimum fin configuration per unit volume. The optimization procedure is also performed to

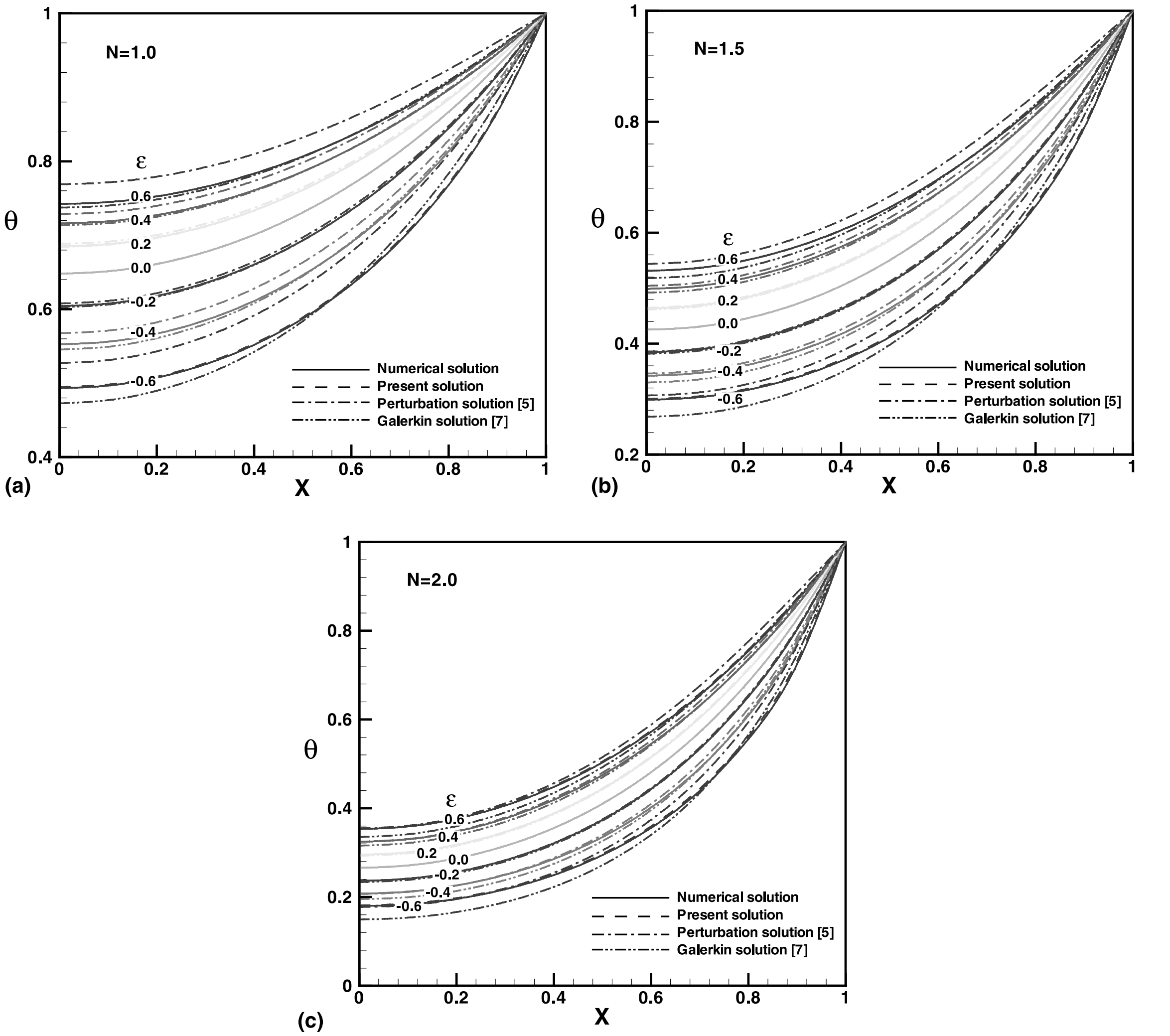

Fig. 5. Temperature distribution in convecting fins with variable thermal conductivity correspond to: (a) $N = 1.0$; (b) $N = 1.5$; (c) $N = 2.0$.

establish the profile area A_p by first expressing Q_n/B as a function of N (or b) and then searching for the optimum value of N (or b).

8. Results

The values of coefficient C relative to the thermal conductivity parameters, ε, and the specified fin parameters, N, are calculated and indicated in Fig. 4. The profiles show that the characteristic value of the temperature at the fin tip increases with increased thermal conductivity of the fin. However, as expected, the tip temperature decreases with increasing fin length, with the rate of decrease reducing as the fin length increases.

The nondimensional temperature distributions along the fin surface with ε varying from -0.6 to 0.6 are depicted in Figs. 5(a)–(c) for different values of $N = 1.0$, 1.5, and 2.0, respectively. The figures also present a comparison of the nondimensional temperature distributions obtained from the decomposition method with those obtained from the numerical solution, the perturbation method, and the Galerkin approximate method. It will be seen that if the thermal conductivity of the fin's material increases with temperature ($\varepsilon > 0$), the mean temperature increases. Conversely, if the thermal conductivity decreases with temperature ($\varepsilon < 0$), the result is a decrease in the mean temperature. This is a consequence of the nonlinearity which is associated with temperature-dependent thermal conductivity conditions, and which is absent for constant thermal conductivity fins. It is observed that the decomposition method results match almost exactly with the numerical solution for all values of ε and N. In fact, the comparison is remarkably good since for $\varepsilon = -0.6$ and $N = 2.0$, the largest error observed never exceeds 0.2%. By comparison, the perturbation solution and the Galerkin solution yield significant error for values of ε exceeding ± 0.4. Regarding the Galerkin solution, in order to yield higher accuracy for larger values of $|\varepsilon|$, it is necessary to modify the trial function to allow greater flexibility in approximating the true solution.

Fig. 6 presents the fin efficiency relative to ε for specified values of N. For a given value of N, it will be seen that the efficiency, η, increases as ε increases. This may be explained by the fact that the thermal conductivity increases with temperature ($\varepsilon > 0$), causing the heat transfer rate to increase. The corresponding curves obtained by numerical integration are plotted on the same figure for comparison purposes. The results derived by the Adomian decomposition method correspond exactly with the results of the numerical solutions.

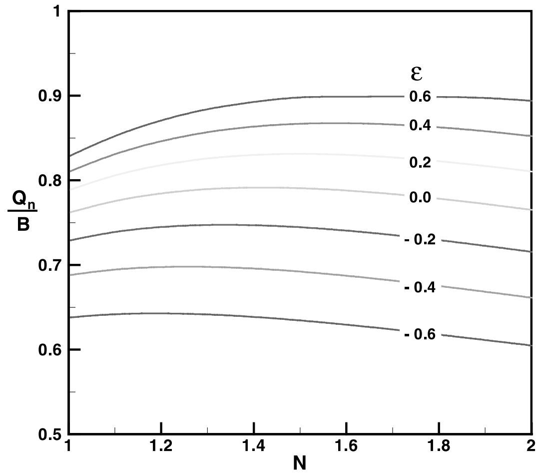

Fig. 7. The dimensionless heat transfer, Q_n/B, as function of N with variable thermal conductivity.

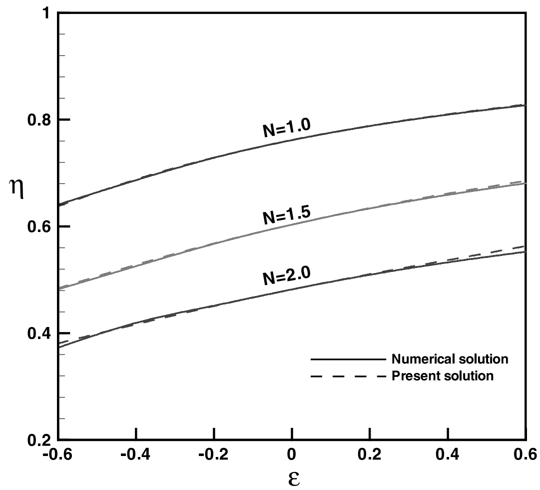

Fig. 6. Efficiency of convecting fins.

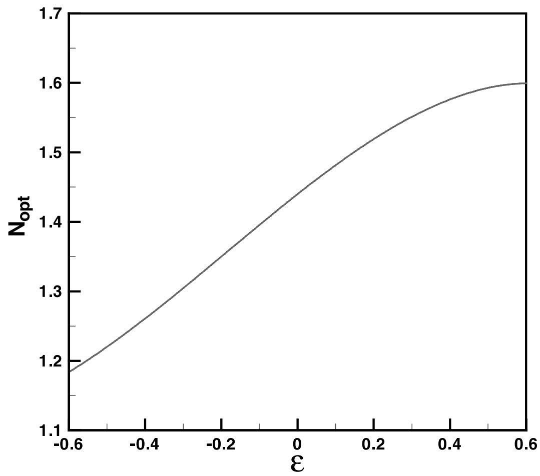

Fig. 8. Relationship between optimum N and ε.

The presented results indicate that for $\varepsilon = -0.6$ and $N = 0.2$ the maximum error is about 0.01.

The nondimensional heat transfers Q_n/B (for a unit fin volume) varying with N from 1 to 2 are depicted in Fig. 7. For specified values of ε, under a given profile area, A_p, the heat transfer first rises and then falls as the fin length increases. In general the optimum fin length (at which Q_n/B reaches a maximum value) increases as ε increases. The optimum value of N can be obtained based upon the value of ε. Therefore, the optimum dimensions of the convective fin with variable thermal conductivity may be established. The relative values of optimum N and ε are shown in Fig. 8.

9. Conclusions

A nonlinear, convective, rectangular fin with variable thermal conductivity has been analyzed using the Adomian decomposition technique, in which the nonlinear problems were treated in a manner similar to linear problems. Linearization, approximation, and assumption are unnecessary during the analytic processes of the decomposition method. The decomposition method offers many advantages over other methods, including faster convergence and higher accuracy, and it may be used to solve the problems associated with the complex conditions presented by fin boundaries and geometries.

References

[1] Q.D. Kern, D.A. Kraus, Extended Surface Heat Transfer, McGraw-Hill, New York, 1972.

[2] P. Razelos, K. Imre, The optimum dimension of circular fins with variable thermal parameters, J. Heat Transfer Trans. ASME 102 (1980) 420–425.

[3] H.M. Hung, F.C. Appl, Heat transfer of thin fins with temperature dependent thermal properties and internal heat generation, J. Heat Transfer Trans. ASME 89 (1967) 155–161.

[4] P. Jany, A. Bejan, Ernst Schmidt's approach to fin optimization: an extension to fins with variable conductivity and the design of ducts for fluid flow, Int. J. Heat Mass Transfer 31 (8) (1988) 1635–1644.

[5] A. Aziz, S.M. Enamul Hug, Perturbation solution for convecting fin with variable thermal conductivity, J. Heat Transfer Trans. ASME 97 (1975) 300–301.

[6] R.J. Krane, Discussion on a previously published paper by A. Aziz and S.M. Euq, J. Heat Transfer Trans. ASME 98 (1976) 685–686.

[7] A. Muzzio, Approximate solution for convective fins with variable thermal conductivity, J. Heat Transfer Trans. ASME 98 (1976) 680–682.

[8] G. Adomian, A review of the decomposition method in applied mathematics, J. Math. Anal. Appl. 135 (2) (1988) 501–544.

[9] G. Adomian, On the solution of algebraic by the decomposition method, J. Math. Anal. Appl. 105 (1) (1985) 141–166.

[10] G. Adomian, Nonlinear Stochastic System Theory and Application to Physics, Kluwer Academic Publisher, Dordrecht, 1988.

[11] G. Adomian, R. Rach, On composite nonlinearties and the decomposition method, J. Math. Anal. Appl. 113 (2) (1986) 504–509.

[12] G. Adomian, Solving frontier problems modelled by nonlinear partial differential equations, Comput. Math. Appl. 22 (8) (1991) 91–94.

[13] W. Lau, C.W. Tan, Errors in one-dimensional heat transfer analysis in straight and annular fins, J. Heat Transfer Trans. ASME 95 (1973) 549–551.

[14] G. Adomian, Solving Frontier Problems in Physics: The Decomposition Method, Kluwer Academic Publishers, Dordrecht, 1994.

[15] A. Re'paci, Nonlinear dynamical system: on the accuracy of Adomian's decomposition method, Appl. Math. Lett. 3 (4) (1990) 35–39.

[16] T.M. Shih, in: Numerical Heat Transfer, Springer, New York, 1984, pp. 187–191.

[17] D.A. Anderson, J.C. Tannehill, R.H. Pletcher, in: Computational Fluid Mechanics and Heat Transfer, McGraw-Hill, New York, 1984, pp. 336–341.